EFFECT OF STREAM SWIRLING ON THE VELOCITY AND TEMPERATURE DISTRIBUTIONS IN A ROUND TUBE

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The results of a numerical study of the effect of stream swirling on the laminar flow structure and heat exchange in a round tube are presented.

Problems connected with the study of the effect of stream swirling on the intensification of processes of heat and mass exchange have been attracting the attention of investigators in recent years [1].

The nature of the attenuation of the tangential component of the laminar flow velocity of a liquid along the radius and the length of a tube was studied in [2] by means of linearization of the complete system of Navier–Stokes equations. A critical curve for the formation of unstable vortex motion was found experimentally by the author in the form of the dependence of the Reynolds number on the angular velocity of the liquid at the entrance to the tube.

The numerical methods which presently exist for calculating the flow of a viscous liquid [3] make it possible to solve the problem in a nonlinear formulation and to determine more accurately the effect of swirling of the stream on the hydrodynamics and heat exchange.

We shall examine the following problem. A liquid having a constant axial velocity V over the cross section at the entrance to a tube is twisted like a solid body. The calculation of the velocity profiles in different cross sections along the length of the tube is required.

Let us write the system of Navier–Stokes equations for the steady laminar flow of an incompressible liquid in cylindrical coordinates:

$$\begin{gathered}\rho\left(v_r\frac{\partial v_r}{\partial r}+v_z\frac{\partial v_r}{\partial z}-\frac{v_\varphi^2}{r}\right)=-\frac{\partial p}{\partial r}+\mu\left(\nabla^2 v_r-\frac{v_r}{r^2}\right),\\ \rho\left(v_r\frac{\partial v_\varphi}{\partial r}+\frac{v_r v_\varphi}{r}+\frac{\partial v_\varphi}{\partial z}\right)=\mu\left(\nabla^2 v_\varphi-\frac{v_\varphi}{r^2}\right),\\ \rho\left(v_r\frac{\partial v_z}{\partial r}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\nabla^2 v_z,\\ \frac{\partial v_r}{\partial r}+\frac{v_r}{r}+\frac{\partial v_z}{\partial z}=0.\end{gathered}\tag{1}$$

The boundary conditions are taken as follows:

$$v_r=v_\varphi=v_z=0 \quad \text{at} \quad r=R,\tag{2}$$

$$v_z=V,\ v_r=0,\ v_\varphi=\Omega r \quad \text{at} \quad z=0,\tag{3}$$

$$v_r=v_\varphi=0,\ v_z=2V(1-r^2/R^2) \quad \text{at} \quad z=Z.\tag{4}$$

The condition (4) means that at a large enough distance from the entrance the flow is assumed to be hydrodynamically stabilized and the tangential velocity component is absent.

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Let us introduce the stream function and the vortical stress function, determined from the following equations:

$$v_z = \frac{1}{\rho r}\cdot\frac{\partial\psi}{\partial r}, \quad v_r = -\frac{\partial\psi}{\partial z}\cdot\frac{1}{\rho r}, \quad \omega = \frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r}. \tag{5}$$

By expressing the velocity components v_z and v_r in the system (1) through ψ and ω and reducing it to dimensionless form we obtain

$$\begin{aligned}
&\frac{\mathrm{Re}\,\bar r^2}{4}\left[\frac{\partial}{\partial \bar z}\left(\frac{\bar\omega}{\bar r}\cdot\frac{\partial\bar\psi}{\partial\bar r}\right) - \frac{\partial}{\partial\bar r}\left(\frac{\bar\omega}{\bar r}\cdot\frac{\partial\bar\psi}{\partial\bar z}\right)\right] - \frac{\partial}{\partial\bar z}\left[\bar r^3\frac{\partial}{\partial\bar z}\left(\frac{\bar\omega}{\bar r}\right)\right] \\
&\qquad - \frac{\partial}{\partial\bar r}\left[\bar r^3\frac{\partial}{\partial\bar r}\left(\frac{\bar\omega}{\bar r}\right)\right] - \mathrm{Re}\,\bar r\,\frac{\partial(\bar v_\varphi)^2}{\partial\bar z} = 0, \\
&\qquad \frac{\partial}{\partial\bar z}\left(\frac{1}{\bar r}\cdot\frac{\partial\bar\psi}{\partial\bar r}\right) + \frac{\partial}{\partial\bar r}\left(\frac{1}{\bar r}\cdot\frac{\partial\bar\psi}{\partial\bar r}\right) + \bar\omega = 0, \\
&\frac{\mathrm{Re}}{4}\left[\frac{\partial}{\partial\bar z}\left(\bar r\,\bar v_\varphi\frac{\partial\bar\psi}{\partial\bar r}\right) - \frac{\partial}{\partial\bar r}\left(\bar r\,\bar v_\varphi\frac{\partial\bar\psi}{\partial\bar z}\right)\right] - \frac{\partial}{\partial\bar z}\left[\bar r^3\frac{\partial}{\partial\bar z}\left(\frac{\bar v_\varphi}{\bar r}\right)\right] - \frac{\partial}{\partial\bar r}\left[\bar r^3\frac{\partial}{\partial\bar r}\left(\frac{\bar v_\varphi}{\bar r}\right)\right] = 0,
\end{aligned} \tag{6}$$

where

$$\bar z = \frac{z}{R}, \quad \bar r = \frac{r}{R}, \quad \bar\psi = \frac{2\psi}{\rho V R^2}, \quad \bar\omega = \frac{2R\omega}{V},$$

$$\bar v_\varphi = v_\varphi/V, \quad \mathrm{Re} = 2R\rho V/\mu.$$

In addition, we introduce the following dimensionless values:

$$\bar v_z = v_z/V, \quad \bar v_r = v_r/V, \quad K = \Omega R/V.$$

To simplify the notation we will henceforth omit the upper bars over the expressions.

The boundary conditions take the following form:

$$\begin{aligned}
&\psi = 1, \quad v_\varphi = 0 && \text{at} \quad r = 1; \\
&\psi = v_\varphi = 0 && \text{at} \quad r = 0; \\
&\psi = r^2, \quad \omega = 0, \quad v_\varphi = Kr && \text{at} \quad z = 0; \\
&\psi = 2r^2 - r^4, \quad \omega = 8r, \quad v_\varphi = 0 && \text{at} \quad z = Z.
\end{aligned} \tag{7}$$

The difference method proposed by the authors of [4], which has sufficient simplicity, economy, and universality, is used for the solution of the system (6).

Each equation of the system (6) can be represented in the form

$$a_\varphi\left[\frac{\partial}{\partial z}\left(\varphi\frac{\partial\psi}{\partial r}\right) - \frac{\partial}{\partial r}\left(\varphi\frac{\partial\psi}{\partial z}\right)\right] - \frac{\partial}{\partial z}\left[b_\varphi r\frac{\partial(c_\varphi\varphi)}{\partial z}\right] - \frac{\partial}{\partial r}\left[b_\varphi\frac{\partial(c_\varphi\varphi)}{\partial r}\right] + rd_\varphi = 0, \tag{8}$$

where φ takes the following values: ω/r, ψ, rv_φ;

$$a_\varphi : \frac{\mathrm{Re}\,r^2}{4}, \; 0, \; \frac{\mathrm{Re}}{4}; \quad b_\varphi : r^2, \; 1/r^2, \; r^2;$$

$$c_\varphi : 1, \; 1, \; 1/r^2; \quad d_\varphi : \mathrm{Re}\,\frac{\partial(v_\varphi^2)}{\partial z}, \; : \; \frac{\omega}{r}, \; 0.$$

The boundary condtions for the vortical stress are found by analogy with [4] from the expressions

$$\begin{aligned}
&\frac{n^3}{6}\cdot\frac{\partial\omega}{\partial n} + \frac{n^2\omega}{2} = 1 - \psi \quad \text{as} \quad r \to 1; \\
&\omega = 8br, \quad \psi = ar^2 + br^4 \quad \text{as} \quad r \to 0,
\end{aligned} \tag{9}$$

where n is the distance along the normal to the boundary surface. The finite-difference approximation (8) is a system of nonlinear algebraic equations which was solved numerically by Seidel's method.

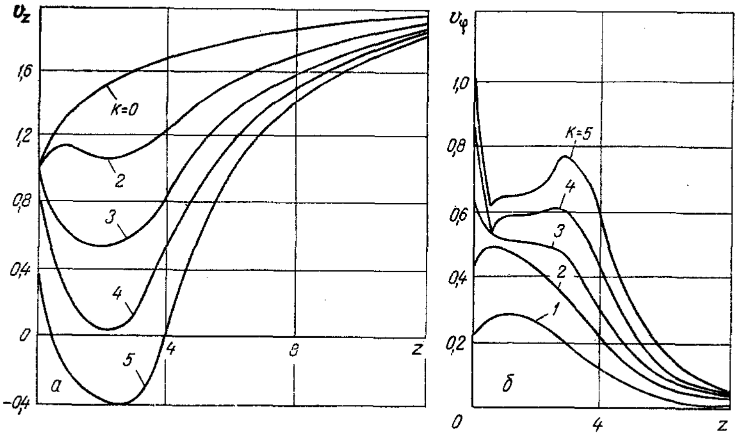

Fig. 1. Velocity distribution along axis of tube (a) and tangential velocity component in initial section of tube at r = 0.21 (b).

A nonuniform 21 × 15 grid is used in the work; Z = 100.

The value

$$\lambda = \left[\frac{\varphi^{(N)} - \varphi^{(N-1)}}{\varphi^{(N)}}\right]_{\max} < 0{,}005, \tag{10}$$

where N is the number of the iteration, served as the convergence criterion.

The axial and radial velocity components were found from the equations

$$v_z = \frac{1}{2r}\cdot\frac{\partial\psi}{\partial r}, \quad v_r = -\frac{1}{2r}\cdot\frac{\partial\psi}{\partial z}. \tag{11}$$

In order to bring out the effect of the degree of swirling of the stream on the velocity field a comparison was made of the data for the Reynolds number Re = 160; the picture is not altered qualitatively for other Reynolds numbers, although the extrema of the functions are displaced both along the length and along the radius of the tube.

The same 21 × 15 grid with Re = 160, Z = 50, and K = 4 was used as a control. The maximum discrepancy proved to be at the entrance; at z = R it was 4% for the axial velocity and 1.5% for the tangential velocity. The error decreases for z > R.

The dependence v_z = f(z, K) at the axis of the tube is presented in Fig. 1a. In the absence of stream swirling (K = 0) we obtain the well-known velocity profile of [5]. At K = 1 the effect of the swirling is slight. At K = 2 the curve has two extrema, which is explained by the complicated nature of the interaction of the frictional forces and the centrifugal forces. For K > 3 the streamlines are displaced toward the walls and the velocity at the axis is decreased at the entrance due to the centrifugal forces. Further along the length of the channel the frictional forces predominate and the velocity begins to increase. At K = 4 and z = 2 the velocity at the axis is decreased to 0, while return currents appear for K > 4. It should be noted that the minimum value of the velocity for different degrees of swirling at Re = const lies at about the same distance from the entrance, and the length of the unstabilized section does not vary significantly with an increase in K.

The radial velocity component v_r comprises a few percent of V in the entire region except for the entrance section (0 < z < 0.6), with $v_r > 0$ in the region of the considerable effect of the centrifugal forces and $v_r < 0$ farther downstream.

For the tangential velocity component the displacement of the maximum toward the axis is characteristic, with v_φ dying down at once for all K in the vicinity of the wall whereas closer to the axis the tangential velocity along the length of the tube (Fig. 1b) differs markedly with a change in K.

In Fig. 2 we present the dependence on z and K of the total shear stress at the wall

$$\tau_f' = \frac{4}{\mathrm{Re}}\sqrt{\left(\frac{\partial v_z}{\partial r}\right)^2 + \left(\frac{\partial v_\varphi}{\partial r}\right)^2}$$

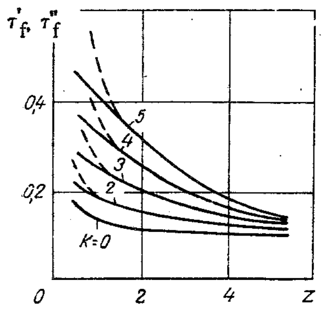

Fig. 2

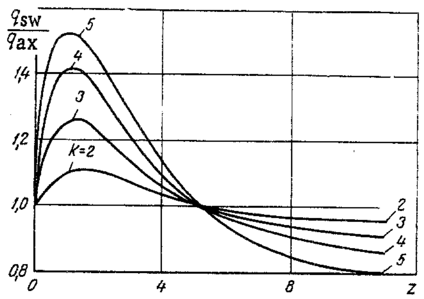

Fig. 3

Fig. 2. Variation of the total shear stress at the wall along the length of the tube (the total shear stress is denoted by a dashed line and the axial component of the shear stress by a solid line).

Fig. 3. Ratio of heat fluxes at wall for swirled and axial flows along length of tube.

(dashed lines) and of the axial component of the shear stress

$$\tau''_f = \frac{4}{\text{Re}}\left(\frac{\partial v_z}{\partial r}\right).$$

It is seen that the pure contribution of the tangential component of the stress is appreciable only at the entrance ($z < 1.5$), and at the same time the redistribution of the axial velocity along the radius considerably increases the shear stress.

It is interesting to study the effect of stream swirling on the heat exchange. Assuming that at the entrance to the tube the liquid has the temperature T_1, the wall temperature is T_2, and the Prandtl number is equal to 1, and neglecting the heating of the liquid due to friction, we obtain the following differential equation:

$$\frac{\text{Re}}{4}\left[\frac{\partial}{\partial z}\left(T\frac{\partial\psi}{\partial r}\right)-\frac{\partial}{\partial r}\left(T\frac{\partial\psi}{\partial z}\right)\right]-\frac{\partial}{\partial z}\left(r\frac{\partial T}{\partial z}\right)-\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=0. \quad (12)$$

The boundary conditions are:

$$T=1 \text{ at } r=1;\quad T=0 \text{ at } z=0;$$
$$\frac{\partial T}{\partial r}=0 \text{ at } r=0;\quad T=1 \text{ at } z=Z, \quad (13)$$

where $T = \tilde{T}-T_1/T_2-T_1$ and $\tilde{T}$ is the current value of the temperature in dimensional form.

Equation (12) was solved similarly to (6), with the value calculated earlier for the stream function being used in this case since in the given formulation the stream function does not depend on the temperature.

The boundary condition is actually not satisfied at the exit from the tube, although the temperature field is little sensitive to this boundary condition, as calculations showed.

A comparison of the heat fluxes for different degrees of swirling is shown in Fig. 3. It is seen that in the region of the considerable effect of centrifugal forces the heat flux into the liquid increases with an increase in K, while farther downstream the opposite effect occurs.

In the case of the formation of return currents (the curve for K = 5, Fig. 1) the minimum temperature over the cross section of the tube appears not in the region of the axis but at r = 0.4, with this effect being observed in the region of $0 < z < 4$. For a comparison with the theoretical and experimental results of [2] a Poiseuille profile was assigned for the axial velocity component at the entrance. The vortex instability observed in [2] was detected by numerical methods at the same values of Re and K, with the first signs of such an instability being observed in the form of individual velocity pulsations at smaller K for the same Re. With the appearance of return currents the condition (10) is not satisfied, although with an increase in the number of iterations the process does not diverge but instead λ varies periodically. In [2] the tangential velocity profile was calculated from the equation

$$v_\varphi(r, z) = v_\varphi(r, 0)\exp(-\beta z).$$

It is seen from Fig. 1b that for $z < 4$ this equation does not correspond to reality, although (14) does not hold for $z > 4$ and the β calculated for our tangential velocity field agree well with the data of [2].

NOTATION

$\nabla^2 = (\partial^2/\partial r^2) + (1/r)(\partial/\partial r) + (\partial^2/\partial z^2)$;

r, z	are the radial and axial coordinates.
v_r, v_φ, v_z	are the velocity components in the radial, tangential, and axial directions;
R	is the radius of the tube;
Ω	is the angular velocity of liquid at entrance;
Z	is the length of tube;
V	is the mean flow rate velocity;
ρ	is the density;
μ	is the dynamic viscosity coefficient.

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